

**Ripassare:**

- **Equazioni, Disequazioni di primo grado e secondo**
- **Piano cartesiano (la retta)**
- **Equazioni e Disequazioni Fratte di secondo grado**
- **Scomposizioni**
- **Equazioni esponenziali**

**Risolvi gli esercizi inserendo sempre le regole che utilizzi, ad esempio la formula risolutiva della equazione di secondo grado**

**Per chi deve Rafforzare la Preparazione gli esercizi da eseguire sono invece il doppio di quelli indicati. Se possibile rivedere ed eventualmente rielaborare le regole inserite nel drive.**

**Esercizi: Piano Cartesiano** (svolgere tutti gli esercizi)

- A) Calcolare la retta fra due punti per le seguenti coppie di punti e determinare il coefficiente angolare della retta:
- $A(-1;3)$  e  $B(0;2)$
  - $A(-2;0)$  e  $B(1;-2)$
  - $A(1;1)$  e  $B(0;-1)$
- B) Disegnare nel piano sia il punto che la retta di seguito e
- $A(-1;3)$  e  $y=x+1$
  - $A(0;3)$  e  $y=2x-1$
  - $A(-1;0)$  e  $y=-x+3$
  - $A(1;1)$  e  $y=-2x+1$
- C) Disegnare le rette e calcolare l'intersezione delle coppie di rette di seguito
- $y=2x+3$  e  $y=x+1$
  - $-3x-y+1=0$  e  $y=2x-1$
  - $y=x-4$  e  $y=-x+3$
  - $y=x$  e  $y=-2x+1$

**Esercizi: Equazioni Secondo Grado** (svolgere almeno 10 esercizi fra quelle sottostanti  
6 equazioni e 6 disequazioni)

$$(2x + 1)^2 - x^2 - (x - 1)^2 = (2x + 3)(2x - 3) + 1 \quad [-1; 4]$$

$$(2 - 3x)(x - 2) + 3(x - 1)^2 = (x - 1)(x + 3) \quad [\pm \sqrt{2}]$$

$$(1 - x)^2 = 2x + \frac{x^2 - 3x + 7}{2} \quad \left[ \frac{5 \pm 3\sqrt{5}}{2} \right]$$

$$\frac{2}{3}x + \frac{1}{2}x(x + 2) - 5x + \frac{1}{6} = \frac{x}{3}(x - 5) \quad [5 \pm 2\sqrt{6}]$$

$$(2 - 3x)^2 - (2x + 1)^2 = 4(2 - 4x) \quad [\pm 1]$$

$$x - (2x - 1)^2 + \frac{1}{2} = \frac{3 - x}{2} - (3x - 1)(x - 2) \quad \left[ 0; -\frac{3}{2} \right]$$

$$(3x - 4)^2 - 3x^2 = 2(8 + 13x) \quad \left[ 0; \frac{25}{3} \right]$$

$$x(x + 2\sqrt{2}) + 2\sqrt{3}x(1 + \sqrt{2}) = 2x(\sqrt{3} + \sqrt{2}) \quad [0; -2\sqrt{6}]$$

$$\left(x + \frac{2}{3}\right)\left(x - \frac{2}{3}\right) - 2x(x - 1) = 2\left(x - \frac{2}{3}\right) \quad \left[ \pm \frac{2\sqrt{2}}{3} \right]$$

$$x(1 - 5x) + 3 = [5 - (2 + 5x)]x - 2(x + 1) - (x^2 - 6) \quad [\pm 1]$$

$$2(x^2 + 2) - 2(x + 3)(x - 3) - 4 = 7x^2 - (3x + 4)^2 + 34 \quad [0; -12]$$

$$2x(x - 5) - (2x - 3)(x + 1) = x(2 - x) - 15 \quad [2; 9]$$

$$(x + 3)(3x - 1) - 2[2x^2 - x(x - 2)] + 6 = 0 \quad [-3; -1]$$

$$-6x + \left(\frac{1}{2} - x\right)\left(\frac{1}{2} + x\right) - 9(-1)^2 < 0 \quad \left[ x < -\frac{7}{2} \vee x > -\frac{5}{2} \right]$$

$$\frac{1}{5}\left(\frac{x - 2}{2}\right) + \frac{4x^2 + x}{4} - \frac{1}{8}\left(1 + \frac{13}{5}\right) > 0 \quad \left[ x < -1 \vee x > \frac{13}{20} \right]$$

$$\frac{5}{4} + \frac{x}{3}(3x - 8) - \frac{5}{3}\left(\frac{1}{4} - 2x\right) \leq \frac{2}{3}\left(x + \frac{65}{12}\right) \quad \left[ -\frac{5}{3} \leq x \leq \frac{5}{3} \right]$$

$$2\left[\left(-\frac{1}{4}\right)^2 - 4x\right] - \frac{1}{4}(x + 1) - (15 + x^2) > 0 \quad \left[ -\frac{11}{2} < x < -\frac{11}{4} \right]$$

$$\sqrt{3}x^2 - x + \frac{1}{2} \geq \sqrt{3}x - \frac{1}{2} \quad \left[ x \leq \frac{\sqrt{3}}{3} \vee x \geq 1 \right]$$

$$\frac{1 - x + x^2}{2} + \frac{x(3x + 16)}{8} - \frac{3x^2 + 2}{4} \leq x^2 + \frac{5x - 4}{3} \quad \left[ x \leq -\frac{4}{3} \vee x \geq \frac{8}{7} \right]$$

$$\frac{8}{5}\left(\frac{15}{2}x + 90\right) - \frac{6}{5}\left[20x + \frac{6}{5}(-10)^2\right] - x^2 < 0 \quad [x < -12 \vee x > 0]$$

$$\left(-\frac{1}{2}\right)^2 [2 - (-2x)^2] - \frac{2 + 9x}{9} - \frac{1}{2}(x + 1) > 0 \quad \left[ -\frac{4}{3} < x < -\frac{1}{6} \right]$$

$$\quad \quad \quad [x < 0 \vee x > 3]$$

**Esercizi: Disequazioni Fratte Secondo Grado** (svolgere almeno 15 esercizi fra quelle sottostanti)

|                                                              |                                                             |                                            |                                                           |
|--------------------------------------------------------------|-------------------------------------------------------------|--------------------------------------------|-----------------------------------------------------------|
| $\frac{x-1}{x} > 0$                                          | $[x < 0 \vee x > 1]$                                        | $\frac{9x^2 - 12x + 4}{5x^2} \geq 0$       | $[\forall x \in \mathbb{R} - \{0\}]$                      |
| $\frac{x+1}{x} > 0$                                          | $[x < -1 \vee x > 0]$                                       | $\frac{3x^2 + 2}{3x^2 + 2x} \geq 0$        | $\left[x < -\frac{2}{3} \vee x > 0\right]$                |
| $\frac{x}{9x^2 - 6x} > 0$                                    | $\left[x > \frac{2}{3}\right]$                              | $\frac{5 - x^2}{5x + x^2} \leq 0$          | $[x < -5 \vee -\sqrt{5} \leq x < 0 \vee x \geq \sqrt{5}]$ |
| $\frac{x^2 + 4x - 5}{2x - 3} < 0$                            | $\left[x < -5 \vee 1 < x < \frac{3}{2}\right]$              | $\frac{x^2 - 3x + 2}{x^2 + 2x - 8} < 0$    | $[-4 < x < 1]$                                            |
| $\frac{x^2 - 2x + 1}{6x} > 0$                                | $[x > 0, x \neq 1]$                                         | $\frac{x^2 - 3x - 4}{x^2 - 7x + 6} \geq 0$ | $[x \leq -1 \vee 1 < x \leq 4 \vee x > 6]$                |
| $\frac{2x - 8}{(2x - 1)\left(x + \frac{1}{2}\right)} \geq 0$ | $\left[-\frac{1}{2} < x < \frac{1}{2} \vee x \geq 4\right]$ | $\frac{x^2 + 3x + 7}{4x - 4 - x^2} < 0$    | $[\forall x \in \mathbb{R} - \{2\}]$                      |
| $\frac{3}{x - 2} + x + 2 \geq 0$                             | $[-1 \leq x \leq 1 \vee x > 2]$                             | $-\frac{x^2 + 2 - x}{x - 1} + 4 > 0$       | $[x < 1 \vee 2 < x < 3]$                                  |
| $-\frac{2}{x - 3} - x < 0$                                   | $[1 < x < 2 \vee x > 3]$                                    | $\frac{6}{5 - x} \geq x$                   | $[x \leq 2 \vee 3 \leq x < 5]$                            |
| $\frac{x^2 + 1}{2x^2} > 0$                                   | $[\forall x \in \mathbb{R} - \{0\}]$                        | $x \leq \frac{6}{x - 1}$                   | $[x \leq -2 \vee 1 < x \leq 3]$                           |

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|                                                                    |                                                                   |
|--------------------------------------------------------------------|-------------------------------------------------------------------|
| $\frac{1}{3x - x^2} - \frac{4}{x^2 - 6x + 9} \leq \frac{1}{x - 3}$ | $\left[\frac{1}{2} < x < \frac{3}{2} \vee x > \frac{7}{2}\right]$ |
| $\frac{12}{2x - 7} + x + \frac{3}{2} > 0$                          | $[x < -2 \vee -1 \leq x \leq 2]$                                  |
| $3 - x \geq \frac{4}{x + 2}$                                       | $[x < -3 \vee -1 < x < 2]$                                        |
| $4 - x > \frac{10}{x + 3}$                                         | $\left[-\frac{1}{2} < x < \frac{1}{2} \vee x > 3\right]$          |
| $\frac{35}{4(x - 3)} + x + 3 > 0$                                  | $[x \leq -2 \vee 2 \leq x < 3]$                                   |
| $x + 3 \leq \frac{5}{3 - x}$                                       | $\left[-1 < x < \frac{1}{4} \vee x > \frac{2}{3}\right]$          |
| $\frac{7 + 12x}{12} > \frac{25}{12(4x - 1)}$                       | $\left[x < -\frac{3}{2} \vee -\frac{1}{2} < x < \dots\right]$     |
| $\frac{4(x - 4)(x + 6) + 99}{4x - 16} < 0$                         | $[-3 < x < -2 \vee -1 < x < \dots]$                               |
| $\frac{6 + x}{x} < \frac{2}{x + 1}$                                | $\left[\frac{1}{2} \leq x \leq \frac{2}{3} \vee x > \dots\right]$ |
| $-\frac{5}{6x - 9} \leq x + \frac{1}{3}$                           |                                                                   |

**Esercizi: Scomposizioni** (svolgere almeno 20 esercizi fra quelle sottostanti, comunque quante più possibili in preparazione per il programma di 5a)

|                          |                           |                                        |                                           |
|--------------------------|---------------------------|----------------------------------------|-------------------------------------------|
| $x^2 - 49y^2;$           | $9 - a^2b^2.$             | <b>94</b> $2a^4 - 8;$                  | $3x^3y^2 - 27x.$                          |
| $4x^2 - 9y^2;$           | $25a^6b^8 - \frac{1}{4}.$ | <b>95</b> $121y^8 - 49x^2y^4;$         | $x^3 - \frac{49}{9}a^2x.$                 |
| $81 - a^2;$              | $16x^2 - a^4.$            | <b>96</b> $\frac{5}{4}b - 125b^3;$     | $\frac{3}{2}x^5y^3 - \frac{27}{8}x^3y^5.$ |
| $a^4 - 16b^2;$           | $-y^2 + 64x^6.$           | <b>97</b> $(ab - 1)^2 - 1;$            | $25(a - b)^2 - 16(a + b)^2.$              |
| $a^2 - (a + b)^2;$       | $x^6 - 4.$                | <b>98</b> $(a + 3b)^2 - 25b^2;$        | $25x^6 - \frac{1}{9}x^2y^4.$              |
| $x^4 - 49;$              | $9 - 4y^2.$               | <b>99</b> $(-3x - 2)^2 - (-5x + 2)^2;$ | $-\frac{16}{9}x^{10} + 1.$                |
| $16a^2 - 1;$             | $9x^6 - 4.$               | <b>100</b> $x^{2n} - 1;$               | $a^{2n} - b^{6n}.$                        |
| $\frac{1}{81} - x^2y^4;$ | $4a^8 - \frac{1}{16}.$    | <b>101</b> $9x^{2n} - 4y^{8n};$        | $\frac{1}{16}a^{16n} - 25b^{10n}.$        |

|                                   |                                   |                                      |                                                                                          |
|-----------------------------------|-----------------------------------|--------------------------------------|------------------------------------------------------------------------------------------|
| $\frac{2x - 2y}{y - x};$          | $\frac{x^2 - x}{x - 1};$          | $\frac{x^2 + 3x}{3x};$               | $\left[-2; x; \frac{x+3}{3}\right]$                                                      |
| $\frac{2a - 2}{5a - 5};$          | $\frac{-x - y}{x + y};$           | $\frac{4x^2 - 2x}{4x^2};$            | $\left[\frac{2}{5}; -1; \frac{2x-1}{2x}\right]$                                          |
| $\frac{6a - 3b}{6a};$             | $\frac{2x - 2y}{2x + 2y};$        | $\frac{a + b}{a^2 + ab};$            | $\left[\frac{2a-b}{2a}; \frac{x-y}{x+y}; \frac{1}{a}\right]$                             |
| $\frac{a^2 - 2a}{a - 2};$         | $\frac{x}{2x^2 - x};$             | $\frac{x^3 - x^2}{4x^2y};$           | $\left[a; \frac{1}{2x-1}; \frac{x-1}{4y}\right]$                                         |
| $\frac{y - y^2}{a - ay};$         | $\frac{ax}{x^2 - ax};$            | $\frac{4xy}{2x^2y - 2xy};$           | $\left[\frac{y}{a}; \frac{a}{x-a}; \frac{2}{x-1}\right]$                                 |
| $\frac{a + ax}{y + xy};$          | $\frac{2a^2 - 10a}{ax - 5x};$     | $\frac{xy}{x^2 - xy};$               | $\left[\frac{a}{y}; \frac{2a}{x}; \frac{y}{x-y}\right]$                                  |
| $\frac{6x^2 - 12x + 6}{x^2 - 1};$ | $\frac{xy^2 - x}{1 - y};$         | $\frac{4a^2 - 4}{2a + 2};$           | $\left[\frac{6(x-1)}{x+1}; -x(1+y); 2(a-1)\right]$                                       |
| $\frac{6ax - 6a^2}{x^2 - a^2};$   | $\frac{12a - 3a^2}{4y - ay};$     | $\frac{2x^6 + 2x^2}{2x^2};$          | $\left[\frac{6a}{x+a}; \frac{3a}{y}; x^4 + 1\right]$                                     |
| $\frac{-24abx}{8ax - 12bx};$      | $\frac{4a^2 - b^2}{b^3 - 2b^2a};$ | $\frac{x^3 - y^3}{x^3y^3};$          | $\left[\frac{-6ab}{2a-3b}; -\frac{2a+b}{b^2}; \text{non semplif.}\right]$                |
| $\frac{a^3 - 8}{a^2 + 2a + 4};$   | $\frac{a^2 - 5a + 6}{2a - 6};$    | $\frac{3xy + 3y^2}{x^2 - y^2};$      | $\left[a - 2; \frac{a-2}{2}; \frac{3y}{x-y}\right]$                                      |
| $\frac{3x^5}{12x + 12x^2};$       | $\frac{8a^3 + 4a}{4a^2 + 1};$     | $\frac{b^2 + 3b + 2}{b^2 + 2b + 1};$ | $\left[\frac{x^4}{4(1+x)}; \text{non semplif.}; \frac{b+2}{b+1}; \frac{y-3}{y-1}\right]$ |
|                                   |                                   | $\frac{y^2 - 2y - 3}{y^2 - 1};$      | $\left[\frac{x^4}{4(1+x)}; \text{non semplif.}; \frac{b+2}{b+1}; \frac{y-3}{y-1}\right]$ |

|                                                       |                                               |                                               |                                                                                     |
|-------------------------------------------------------|-----------------------------------------------|-----------------------------------------------|-------------------------------------------------------------------------------------|
| $\frac{4x^2 - 4x + 1}{2ax + 2x - a - 1};$             | $\frac{x^2 - 4}{x^2 - x - 2};$                | $\frac{a^4 - 16}{2a^2 + 8}.$                  | $\left[ \frac{2x-1}{a+1}; \frac{x+2}{x+1}; \frac{a^2-4}{2} \right]$                 |
| $\frac{ax - x + 3a - 3}{x^2 + 4x + 3};$               | $\frac{y^2 - 9}{y^3 - 3y^2};$                 | $\frac{x^3 + 4x^2 + 4x}{4 - x^2}.$            | $\left[ \frac{a-1}{x+1}; \frac{y+3}{y^2}; \frac{x(x+2)}{2-x} \right]$               |
| $\frac{x^2y - 4y}{-2y - xy};$                         | $\frac{a^2 - 10a + 25}{a^2 - 25};$            | $\frac{6x^3 - 6xy^2}{x^2 + xy}.$              | $\left[ 2-x; \frac{a-5}{a+5}; 6(x-y) \right]$                                       |
| $\frac{2a^2 - a^2x}{4y - 2xy};$                       | $\frac{6x^2y - 12xy}{9x^4y - 18x^3y};$        | $\frac{x^3 - a^3}{3x - 3a}.$                  | $\left[ \frac{a^2}{2y}; \frac{2}{3x^2}; \frac{x^2 + ax + a^2}{3} \right]$           |
| $\frac{2a^4 - 2x^4}{4a^2 + 4x^2};$                    | $\frac{x^2 - 2x}{x^2 + 2x - 8};$              | $\frac{6ax^2 - 6ay^2}{3x^3 - 3y^3}.$          | $\left[ \frac{a^2 - x^2}{2}; \frac{x}{x+4}; \frac{2a(x+y)}{x^2 + xy + y^2} \right]$ |
| $\frac{4a^3 - 4a^2}{a^3 - 2a^2 + a};$                 | $\frac{(x-2)^2 + 2x}{2(x^3 + 8)};$            | $\frac{30a^2b^2}{a^2b - ab^2}.$               | $\left[ \frac{4a}{a-1}; \frac{1}{2(x+2)}; \frac{30ab}{a-b} \right]$                 |
| $\frac{x^2 - 1}{(x+1)(x^2 - 2x + 1)};$                | $\frac{a^3 - b^3}{a^2 - b^2};$                | $\frac{x^3 - 2x^2 + x}{x^3 - 3x^2 + 3x - 1}.$ | $\left[ \frac{1}{x-1}; \frac{a^2 + ab + b^2}{a+b}; \frac{x}{x-1} \right]$           |
| $\frac{y^2 - 3y + 2}{y^2 - y - 2};$                   | $\frac{16a^2 - 24ab + 9b^2}{28ab - 21b^2};$   | $\frac{1 - 4b^2}{1 + 4b + 4b^2}.$             | $\left[ \frac{y-1}{y+1}; \frac{4a-3b}{7b}; \frac{1-2b}{1+2b} \right]$               |
| $\frac{x^2 + 10xy + 25y^2}{x^4 + 10x^3y + 25x^2y^2};$ | $\frac{a^5 - 16a}{a^3 - 4a};$                 | $\frac{a^2 + ab - 2b^2}{8a^2 - 32b^2}.$       | $\left[ \frac{1}{x^2}; a^2 + 4; \frac{a-b}{8(a-2b)} \right]$                        |
| $\frac{4x^2 + 3x}{4x^3 - x^2 - 3x};$                  | $\frac{2x^2 + x - 10}{(x^2 - 4)(2x^2 + 5x)};$ | $\frac{y^3 - 9y}{y^3 + 4y^2 + 3y}.$           | $\left[ \frac{1}{x-1}; \frac{1}{x(x+2)}; \frac{y-3}{y+1} \right]$                   |

**Esercizi: equazioni esponenziali** (svolgere almeno 6 esercizi fra quelle sottostanti)

$$5^{2x+1} = 25$$

$$7^{5x-2} = -49$$

$$4^{2x+1} = 8^{x+3}$$

$$4^x \cdot 4^{2-5x} = 64 \cdot 16^{3x}$$

$$\left(\frac{1}{3}\right)^{2x+1} \cdot 2^{2x+1} = \frac{3}{2}$$

$$3^{x^2+5x} = \frac{1}{81}$$

$$3^{x^2} = 3$$

$$(3^x)^2 = 3$$

$$4 \cdot 2^x = 8^x$$

$$9^{2x+1} = 27^{x-5}$$

**Esercizi: Frazioni Algebriche** (svolgere almeno 8 esercizi fra quelle sottostanti)

$$\frac{3x}{x^3 - x^2 - x + 1} - \frac{3x}{2 - 2x^2} + \frac{x - 2}{4x - 2x^2 - 2}$$
$$\left(\frac{x^2}{y^2} + \frac{x}{y} + 1\right)\left(\frac{x^2}{y^2} - \frac{x}{y} + 1\right)$$
$$\left[\frac{2(x-y)^2}{(x+y)} \cdot \frac{28}{3(y-x)} \cdot \left(-\frac{x+y}{8}\right) - \frac{x-y}{3}\right] : (x-y)$$
$$\frac{1}{x-y} - \frac{x-y}{x^2 + xy + y^2} + \frac{y^2}{y^3 - x^3}$$
$$\left(1 + \frac{1}{a^2 + a}\right)\left(1 + \frac{1}{a^2 - a}\right) : \left(\frac{a+1}{a^2} + 1\right) : \frac{1}{a^2 - 1}$$
$$\frac{5}{a^2 - 2a + 1} - \frac{a^2 - 14a - 7}{2a^2 - a^4 - 1} - \frac{2}{a^2 + 2a + 1}$$
$$\frac{x^2 + 5x + 4}{x^2 + 7x + 12} : \frac{2x + 2}{3x + 9} : \frac{14x + 14y}{9}$$
$$\frac{x^2 + 5x + 4}{x^2 + 7x + 12} : \left(\frac{2x + 2}{3x + 9} : \frac{14x + 14y}{9}\right)$$
$$\frac{2a}{a^2b - b^3} - \frac{a - b}{2a^2b + 2ab^2} - \frac{a + b}{2a^2b - 2ab^2}$$
$$\left(\frac{y}{x^2 + xy} + \frac{x^2 + xy}{x^2y - y^3} - \frac{1}{x}\right)\left(1 - \frac{2y^2}{x^2 + y^2}\right)$$
$$\left[\frac{x+1}{(x-1)^2}\right]$$
$$\left[\frac{x^4 + x^2y^2 + y^4}{y^4}\right]$$
$$\left[\frac{y(3x-y)}{x^3 - y^3}\right]$$
$$[a^2 - a +$$
$$\left[\frac{4}{(a+1)(a-1)}\right]$$
$$\left[\frac{27}{28(x+y)}\right]$$
$$\left[\frac{7}{3}(x+y)\right]$$
$$\left[\frac{1}{ab}\right]$$
$$\left[\frac{1}{y}\right]$$